SCHWARTZSCHILD RADIATIVE TRANSFER SOLUTIONS IN THE ABSORPTION BAND SATURATION LIMIT

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ABSTRACT. We construct a simple, one-dimensional model of the atmosphere containing a greenhouse gas of sufficiently high density for saturation of an absorption band. This model assumes a given temperature T_S at the surface to account for IR radiation emanating from the surface along with non-radiative mechanisms that affect temperature. The spectral intensity function $I(\nu, z)$ is initialized at the surface ($z = 0$) as $I(\nu, 0) = B(\nu, T_S)$ where ν is the frequency of the absorption band and $B(\nu, T)$ is the spectral density function from Planck's Law. The Schwartzschild Radiative Transfer equation is then used to obtain $I(\nu, z)$ for arbitrary altitude z assuming a given greenhouse gas density $n(z)$. It is shown that in the absorption band *saturation* limit where the greenhouse gas density is sufficiently high for absorption to occur on a much shorter scale length than variations in density with altitude, the spectral intensity $I(\nu, z)$ is merely the thermal emission $B(\nu, T(z))$ at altitude z, and is not affected by low altitude emissions. Therefore, energy balance at high altitudes is possible in the band saturation limit while surface temperatures are insensitive to increases in the greenhouse gas density.

1. INTRODUCTION

The issue of absorption saturation of the 15μ band of carbon dioxide (CO_2) has been a rather hotly debated topic over the last few decades. The experiments by Knut Ångström and J. Koch (Ref. 1) in 1901 seemed to show that while CO_2 strongly absorbs 15μ radiation, that absorption *saturates* at higher CO_2 concentrations, and adding more CO_2 does not result in significantly more absorption. This was the prevailing theory for at least thirty years.

Then in 1931, E.O. Hulbert modified the work by Svante Arrhenius (Ref. 2) to include convection in his assumed purely radiometric model. His results predicted that convective equilibrium prevails in the lower troposphere up to an altitude of about 10 km while radiative equilibrium holds above (Ref. 3). Climate scientists often claim this implies that the details of absorption in the lower troposphere do not matter since heat "is spread around and transferred upward by convection", and what governs temperatures at all levels is the *radiative balance*[1] in the upper troposphere where the air is thinner and the 15μ absorption band of CO_2 is far from saturation. Therefore, adding more carbon dioxide at these altitudes would cause more warming. The major irony in this explanation, however, is the fact that the entire objective of anthropogenic global warming (AGW) research is to make predictions about temperature changes *at the surface and lower altitudes* where details of absorption "do not matter"according to their logic.

[1]The radiative energy absorbed by the earth and its atmosphere must equal the amount of such energy emanating from the upper atmosphere in order to have a steady temperature. There would be warming if absorption of radiation exceeded emission, and cooling otherwise.

Another popular argument from the AGW community is that in the upper troposphere where there is still some CO_2, but not much 15μ radiation, the CO_2 molecules can still become energized in collisions with N_2 and O_2 molecules. These excited CO_2 molecules then release this energy as heat in subsequent collisions, causing a rise in temperature (Ref. 4). The fallacy behind this argument is that they ignore the fact that kinetic energy had to be *removed from* the colliding molecules in order for the CO_2 molecule to make the transition to the higher energy state. On a macroscopic scale, this would result in *cooling*. Therefore, any atmospheric warming from these collision-excited CO_2 molecules will be met with an equal amount of cooling, resulting in a zero sum gain in heat energy. This is simply energy conservation. For warming, the atmosphere must capture energy coming in from the outside, primarily upwelling IR radiation from the surface but also some from outer space. Just as a pot of water cannot heat itself, neither can the atmosphere.

For making valid scientific arguments, we need a relatively simple but precise model for tracking spectral intensities, absorption, and saturation. It need not be accurate to multiple decimal places, but must take all dependencies into account in order to show correct trends. Arguments and explanations made "on the fly"often miss one or more important aspects. In the model we suggest, we assume a given temperature profile $T(z)$ which takes into account the effect of all greenhouse gases and non-radiative sources of temperature changes. In doing so, we anticipate such variations to be less than 10°K, which is just a few percent on the absolute temperature scale, Also, we assume a given greenhouse gas density profile $n(z)$ which can be adjusted to study the effects of increasing this gas density. We then use the Schwartzschild Radiative Transfer Equation to see what trend in the spectral intensity $I(\nu, z)$ develops as $n(z)$ increases. In particular, we are interested in the case of what happens to I in the *saturation limit* as $n(z) \to \infty$. According to the logic of the AGW community, the surface of the earth under such an insulating atmosphere would get too warm to sustain life in order to maintain the required radiative energy balance of the earth with the radiation received from the sun. In this paper, however, we show that there would be no connection between the surface and top of the atmosphere (TOA) temperatures. Radiative energy balance can be maintained at the TOA without affecting the surface temperature.

2. Schwartzschild Radiative Transfer Equation

2.1 General Solution. The Schwartzschild Radiative Transfer Equation is given by

$$\frac{\partial}{\partial z} I(\nu, z) \, + \, n(z)\, \alpha(\nu) I(\nu, z) \, = \, n(z)\, \alpha(\nu)\, B(\nu, T(z)) \tag{1}$$

where $I(\nu, z)$ is the spectral intensity of the radiation at frequency and z is the altitude. The temperature $T(z)$ is assumed to be a given function since we do not anticipate changes of more than about 10°K. Also, we are assuming the density and absorptivity profiles $n(z)$ and $\alpha(\nu)$ are given functions. Therefore, this model would not include the pressure broadening effect on the 15μ absorption band which comes from increased CO_2 concentration, but this is relatively minor (1-2%). Using the standard integrating factor method for solving first-order linear differential

equations, we obtain a general solution of

$$I(\nu,z) \;=\; \int_0^z n(z')\alpha(\nu)\,B(\nu,T(z'))\,\exp\!\left[-\alpha(\nu)\int_{z'}^z n(s)\,ds\right]dz' \qquad (2)$$
$$+\; I_S(\nu)\exp\!\left[-\alpha(\nu)\int_0^z n(z')\,dz'\right]$$

where $I_S(\nu)$ is the value of $I(\nu,z)$ at the surface (ie. $z = 0$) which arises as an arbitrary constant when solving equation (1). Since the radiation emanating from the surface is a blackbody at temperature T_S, we have $I_S(\nu) = B(\nu,T_S)$ and equation (2) becomes

$$I(\nu,z) \;=\; \int_0^z n(z')\alpha(\nu)\,B(\nu,T(z'))\,\exp\!\left[-\alpha(\nu)\int_{z'}^z n(s)\,ds\right]dz' \qquad (3)$$
$$+\; B(\nu,T_S)\exp\!\left[-\alpha(\nu)\int_0^z n(z')\,dz'\right]$$

The first term on the right-hand side of equation (4) accounts for contributions of attenuated blackbody radiation in the upward direction from infinitesimal layers (of thickness dz') at altitude $z' < z$. Each such layer radiates with intensity $B(\nu,T(z'))$ which is attenuated over the distance $z - z'$ in reaching altitude z. The integral in this equation indicates the summation of intensity contributions from all such blackbody layers. The second term on the right-hand side of equation (3) is the attenuated radiation intensity from surface emissions. It is the difference between the values of this term at the surface ($z = 0$) and the top of the troposphere that provides the forcing for temperature change. Near the surface, we can choose $n(z)$ such that the CO_2 concentration is 300-400 ppm and $\Delta L \approx 10$ meters.

2.2 Saturation. A condition of saturation arises in equation (2) if we can choose a scale length ΔL such that there is little variation of $n(z)$ and $T(z)$ over an interval of length ΔL, but also $\exp[-\alpha(\nu)\,n(z)\,\Delta L] << 1$. Therefore we write

$$n(s) \approx \text{constant} \qquad \text{for} \qquad z' - \Delta L < s < z' \qquad (4)$$

and

$$\exp[-\alpha(\nu)\,n(z)\,\Delta L] \;<<\; 1 \qquad (5)$$

as our conditions for saturation at altitude z. Near the surface, we can choose $n(z)$ such that the CO_2 concentration is 300-400 ppm and $\Delta L \approx 10$ meters. At higher levels, $n(z)$ would decrease, but larger values of ΔL could also be chosen, thereby compensating for the lower $n(z)$ values. Additionally, it should be pointed out that by far most of the 15μ band absorption of CO_2 takes place within a few km from the surface. Finally, since we are considering the case of spectral density $I(\nu,z)$ in the band saturation limit (ie. the limit as $n \to \infty$) we assume saturation at all altitudes.

We first note that since $z >> \Delta L$ and $n(z)$ varies little over a z interval of length ΔL, the second term on the right-hand side of equation (3) vanishes in the saturation limit. Therefore, we have

$$I(\nu,z) \;=\; \int_0^z n(z')\,\alpha(\nu)\,B(\nu,T(z'))\,\exp\!\left[-\alpha(\nu)\int_{z'}^z n(s)\,ds\right]dz' \qquad (6)$$

in the absorption band saturation limit. From inequality (5), we see that the exponential factor inside the integral becomes negligible for values of $z' < z - \Delta L$.

Therefore, we have

$$I(\nu, z) \;=\; \int_{z-\Delta L}^{z} n(z')\,\alpha(\nu)\,B(\nu, T(z'))\,\exp\!\left[-\alpha(\nu)\int_{z'}^{z} n(s)\,ds\right] dz' \qquad (7)$$

Then, since $n(s)$ varies little over a layer of thickness ΔL (see inequality (7)), we have from equation (7)

$$\begin{aligned} I(\nu, z) \;&\approx\; \int_{z-\Delta L}^{z} n(z')\,\alpha(\nu)\,B(\nu, T(z'))\,\exp\!\left[-\alpha(\nu)\,n(z)\int_{z'}^{z} ds\right] dz' \qquad (8)\\ &=\; n(z)\,\alpha(\nu)\,B(\nu, T(z))\int_{z-\Delta L}^{z} \exp\!\left[-\alpha(\nu)\,n(z)(z-z')\right] dz'\\ &=\; \frac{n(z)\,\alpha(\nu)}{n(z)\,\alpha(\nu)}\,B(\nu, T(z))\left(1 \;-\; \exp[-\alpha(\nu)\,n(z)\,\Delta L]\right) \;\approx\; B(\nu, T(z)) \end{aligned}$$

where we have used approximation (5) in the last step. From this result, we see that under the conditions of absorption band saturation (ie. the greenhouse gas density $n(z)$ is high enough to satisfy inequalities (4) and (5)), the spectral irradiance $I(\nu, z)$ at high altitudes is not affected by corresponding values at the surface or by increasing $n(z)$. This contracts the claim from the AGW community that increasing the greenhouse gas density, even at band saturation, further reduces the outgoing spectral intensity $I(\nu, z)$, thereby causing increased warming at lower altitudes. Therefore, energy balance can be maintained in the upper atmosphere while temperatures near the surface remain nearly constant, and there is no "surface budget fallacy"as claimed by the AGW community (Ref. 5). In the case of CO_2, radiation of wavelength 15μ emanating from the surface is strongly attenuated when it reaches the upper atmosphere, and plays no significant role in attaining energy balance at those altitudes.

3.0 Conclusion

We have shown, applying the Schwartzschild Radiative Transfer Equation to a relatively simple atmospheric model, that a greenhouse gas of sufficiently high density will cause saturation of its absorption band and will strongly attenuate that spectral band before it reaches high altitudes in the troposphere. Therefore, this band saturation does not significantly affect upper atmospheric energy balance. At low altitudes, the absorption band radiation emanating from the surface will continue upward until all available energy from that band has been absorbed. At this point, the surface temperature will not increase by adding more greenhouse gas. The absorptivity of CO_2 at the 15μ band along with its current atmospheric concentration makes it consistent with the requirements for band saturation.

References

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